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Technical Remarks and Comments on the UV/IR-Mixing Problem of a Noncommutative Scalar Quantum Field Theory

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Abstract. In this letter we will dicuss the possibility of a resummation procedure in order to cure the UV/IR-mixing problem of noncommutative field theories. The method is presented for a scalar ϕ^4 theory on Euclidean space. Finally, we sketch the idea of resummation for $U(1)$-gauge theories.

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1 Introduction

In the first part of the present paper we discuss different possibilities to cure the UV/IR-mixing of a scalar noncommutative quantum field theory at the one-loop level. Especially, we investigate the idea of resummation proposed in the literature [1]. In the second part we give some ideas how to import this method onto gauge theories.
In order to describe the noncommutative scalar quantum field theory one generalizes the usual concepts of quantum mechanics, which are defined by the following commutation relation [2], [3],

$$[\hat{X}_\mu, \hat{P}_\nu] = i\delta_{\mu\nu}, \quad [\hat{X}_\mu, \hat{X}_\nu] = [\hat{P}_\nu, \hat{P}_\nu] = 0, \qquad (\hbar = 1) \tag{1}$$

where $\hat{X}_\nu$ and $\hat{P}_\nu$ are the hermitian position and momentum operators, respectively.
However, there is no evidence that these concepts are sufficient at very short distance, implying the following natural generalization

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}, \tag{2}$$

with the deformation parameter $\theta_{\mu\nu}$ being an antisymmetric constant matrix of dimension $[\text{length}]^2$. In order to maintain Lorentz symmetry, the existence of such a constant antisymmetric tensor makes necessary a modification of the Lorentz transformation [4]. To construct the perturbative field theory formulation we will use ordinary fields and not operator-valued objects. One has the so-called Moyal-Weyl correspondence defined by

$$\hat{\phi}(\hat{X}) \Longleftrightarrow \phi(x), \tag{3}$$

where $\hat{\phi}(\hat{X})$ is the operator valued functional and $\phi(x)$ is the usual scalar field depending on ordinary (commuting) Euclidean coordinates x_μ. The correspondence is given by

$$\begin{aligned} \hat{\phi}(\hat{X}) &= \int \frac{d^4k}{(2\pi)^4} e^{ik\hat{X}} \phi(k), \\ \phi(k) &= \int d^4x e^{-ikx} \phi x. \end{aligned} \tag{4}$$

Here k and x are commutative, real variables. With the help of the Baker-Campbell-Hausdorff-formula one finds

$$\begin{aligned} \hat{\phi}_1(\hat{X})\hat{\phi}_2(\hat{X}) &= \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} e^{ip\hat{X}} e^{iq\hat{X}} \phi_1(p)\phi_2(q) \\ &= \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} e^{i(p+q)\hat{X} - \frac{i}{2}\theta_{\mu\nu}p_\mu q_\nu} \phi_1(p)\phi_2(q). \end{aligned} \tag{5}$$

Now one can define the Moyal (or star)-product,

$$\hat{\phi}_1(\hat{X})\hat{\phi}_2(\hat{X}) \Longleftrightarrow \Big(\phi_1 \star \phi_2\Big)(x), \tag{6}$$

which is defined as [2], [5]

$$\Big(\phi_1 \star \phi_2\Big)(x) := \int \frac{d^4k}{(2\pi)^4} \int d^4y \phi_1(x + \frac{1}{2}\tilde{k})\phi_2(x+y)e^{iky}, \quad \tilde{k}_\mu = \theta_{\mu\nu}k_\nu. \tag{7}$$

With the star-product one is able to define the noncommutative scalar self-interacting classical action in a four-dimensional Euclidean space,

$$\Gamma^{(0)}[\phi] = \int d^4x \Big(\frac{1}{2} (\partial_\mu \phi(x) \partial_\mu \phi(x) + m^2 \phi^2(x)) + \frac{g^2}{4!} \phi \star \phi \star \phi \star \phi(x) \Big). \tag{8}$$

Note that the quadratic part of the action (8) remains unchanged compared to the commutative theory. Only the interaction term is modified and therefore the corresponding Feynman-rules for the interaction vertices are changed by additional phase factors. In perturbation theory, the interplay of these phase factors produces two different types of graphs leading to planar and non-planar contributions. These are distinguished by their behaviour with respect to the ultra-violet (UV)-region. The planar graphs show the desired effects expected from naive power counting and known from commutative theory. The planar divergent radiative corrections can therefore be discussed in the framework of the usual renormalization procedure. The second class of graphs, the non-planar ones, show an ugly nonlocal behaviour. The a priori divergent contributions are regularized by the phase factors. Thus, the non-planar graphs are UV-finite, but instead develop a new singularity for small external momenta (if one discusses one-loop self-energy corrections). This artefact is the so called UV/IR-mixing problem of noncommutative quantum field theories [6]. The present work rediscusses this UV/IR-problem within the framework of the resummed theory [1] for a scalar ϕ^4 theory and presents the idea of a resummation of gauge field models.

2 Review of the traditional approach

In order to understand the UV/IR-mixing in the framework of a resummed theory it is useful to show how the UV/IR-problem enters the game [1], [6]. The action (8) induces the following one-loop Feynman integral describing the first order quantum correction to the two point function

$$\Delta\Sigma = \frac{g^2}{6} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m^2} (2 + \cos(k\tilde{p})), \quad \tilde{p}_\mu = \theta_{\mu\nu} k_\nu, \tag{9}$$

where we have used the non-resummed propagator for the scalar field,

$$\Delta(k) = \frac{1}{k^2 + m^2}, \tag{10}$$

and the corresponding Feynman rule for the noncommutative interaction vertex [6]. This integral splits up in a planar contribution (leading to the usual mass renormalization),

$$\Delta\Sigma_p = \frac{g^2}{3} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m^2}, \tag{11}$$

and a non-planar contribution,

$$\Delta\Sigma_{np} = \frac{g^2}{6} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m^2} e^{ik\tilde{p}}. \tag{12}$$

With the usual techniques using Schwinger parametrization one gets for the non-planar expression after Gaussian integration

$$\Delta\Sigma_{np} = \frac{g^2}{6}\frac{\pi^2}{(2\pi)^4}\int_0^\infty \frac{d\alpha}{\alpha^2} e^{-\alpha m^2 - \frac{\tilde{p}^2}{4\alpha}}, \tag{13}$$

where $\tilde{p}^2$ acts as regulator. With $\int_0^\infty \frac{d\alpha}{\alpha^2}\exp(-u\alpha - v/(4\alpha)) = 4\sqrt{(u/v)}K_1(\sqrt{uv})$ for positive real part of u and v we find

$$\Delta\Sigma_{np} = \frac{g^2}{6}\frac{\pi^2}{(2\pi)^4} 4\sqrt{\frac{m^2}{\tilde{p}^2}} K_1\left(\sqrt{m^2\tilde{p}^2}\right). \tag{14}$$

With the expansion of the modified Bessel function $K_1(x) = \frac{1}{x} + \frac{x}{4}(2\gamma - 1 - 2\ln 2) + \frac{x}{2}\ln x + \mathcal{O}(x^3)$ we find for small $\tilde{p}^2$

$$\Delta\Sigma_{np} = \frac{g^2}{24\pi^2}\Big(\frac{1}{\tilde{p}^2} + \frac{m^2}{4}\ln(m^2\tilde{p}^2) + \dots\Big), \tag{15}$$

where the dots stand for the terms remaining finite for $\tilde{p}^2 \to 0$. Thus, as suggested in the introduction, in the non-planar section of noncommutative field theory the original UV-divergence of the commutative theory has been regularized by the momentum dependend cut-off $\tilde{p}^2$. In the commutative limit $\theta_{\mu\nu} = \tilde{p}^2 = 0$ the divergence reappears. Unfortunately, even the regularized divergence causes troubles: The first term on the right hand side of (15) gives rise to severe IR-singularities when inserted into higher order loop integrals $\int d^4p$. This miraculous conservation of misery is called UV/IR-mixing of divergences. After performing the ordinary mass renormalization (treating the planar correction) $M^2 = m^2 + \delta m^2$ the effective two point action to first order is

$$\Gamma_2^{(1)} = \int \frac{d^4p}{(2\pi)^4}\phi(p)\phi(-p)\Big(p^2 + M^2 + \frac{g^2}{24\pi^2}\big(\frac{1}{\tilde{p}^2} + \frac{M^2}{4}\ln(M^2\tilde{p}^2) + \dots\big)\Big). \tag{16}$$

Now, in order to eliminate the UV/IR-mixing, one has to handle the horrible nonlocal $1/\tilde{p}^2$ term. In [7] a field redefinition has been tried. Here we present another approach, a resummation procedure coming from quantum field theory at finite temperature and imported on noncommutative field theories by [1].

3 The resummation theory revisited

The recipe to cure the UV/IR-mixing (at one-loop order) via resummation is given by adding and subtracting to the classical action (8) the term [1],

$$\frac{g^2}{24\pi^2}\int d^4x \frac{1}{2}\phi(x)\frac{1}{\tilde{\partial}^2}\phi(x), \tag{17}$$

implying that one has now the following tree level action,

$$\Gamma_R^{(0)}[\phi] = \int d^4x \Big(\frac{1}{2}(\partial_\mu\phi(x)\partial_\mu\phi(x) + m^2\phi^2(x) - \phi(x)\frac{\tilde{c}}{\tilde{\partial}^2}\phi(x) + \phi(x)\frac{\tilde{c}}{\tilde{\partial}^2}\phi(x)) + \frac{g^2}{4!}\phi\star\phi\star\phi\star\phi(x)\Big), \tag{18}$$

with $\tilde{c} = \frac{g^2}{24\pi^2}$. The idea is to treat one of the two cancelling terms as modification of the propagator and the other as a two-point vertex function. Doing loop-expansion, this leads to a mixing of orders in the coupling constant g^2 (which is exactly the desired effect of the resummation procedure), but non-perturbatively the theory remains the same. The process of resummation allows in principle two possibilities for the resummed propagators,

$$\Delta_{\pm}(k) = \frac{1}{k^2 + m^2 \pm \frac{\tilde{c}}{\tilde{k}^2}}. \tag{19}$$

As argued in [1] the negative sign corresponds to unphysical tachyonic poles. Therefore it seems natural that only the positive sign is meaningful. However, aiming at further calculations concerning gauge theory, we want to stay as general as possible.
Now one can compute the one-loop quantum correction with the resummed propagator,

$$\Delta\Sigma_{\pm} = \frac{g^2}{6} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m^2 \pm \frac{\tilde{c}}{\tilde{k}^2}} (2 + \cos(k\tilde{p})) = \frac{g^2}{6} \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{k}^2}{k^2\tilde{k}^2 + m^2\tilde{k}^2 \pm \tilde{c}} (2 + e^{ik\tilde{p}}). \tag{20}$$

In the UV-region the integral (20) has the same structure as for the non-resummed theory. Thus, we expect a quite similar result, with a quadratically divergent contribution from the planar graph and a finite but nonlocal $\frac{1}{\tilde{p}^2}$-contribution from the non-planar graph. This will be verified by explicit calculation.
In order to get a Gaussian integral after Schwinger parametrization we have to expand (20) into partial fractions. Unfortunately, the term $k^2\tilde{k}^2$ causes troubles unless $\tilde{k}^2 \propto k^2$. Since $\theta_{\mu\nu}$ can always be transformed into a block matrix

$$\theta_{\mu\nu} = \begin{pmatrix} 0 & \theta_{12} & 0 & 0 \\ -\theta_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_{34} \\ 0 & 0 & -\theta_{34} & 0 \end{pmatrix}, \tag{21}$$

we see that $\theta_{\mu\nu}$ has only two degrees of freedom. By eliminating one degree of freedom, thus using the choice $\theta_{12} = \theta_{34} =: \theta$, we find $(\theta^2)_{\mu\nu} = \theta^2 \otimes \mathbb{1}_{4\times 4}$. At least for this special choice we can calculate

$$\begin{aligned} \Delta\Sigma_{\pm} &= \frac{g^2}{6} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2)^2 + m^2k^2 \pm \frac{\tilde{c}}{\theta^2}} (2 + e^{ik\tilde{p}}) \\ &= -\frac{g^2}{6} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{2u} \Big(\frac{1}{k^2 + \frac{m^2}{2} + u} - \frac{1}{k^2 + \frac{m^2}{2} - u} \Big) (2 + e^{ik\tilde{p}}). \end{aligned} \tag{22}$$

Here the (possibly complex) quantity $u = \sqrt{\frac{m^4}{4} \mp \frac{\tilde{c}}{\theta^2}}$. Now we can use formulae (40), (41) from the appendix (if u is real, we introduce a convergence factor $b \to 0$ by hand) and obtain for the non-planar part

$$\begin{aligned} \Delta\Sigma_{np\pm} &= \frac{g^2}{24\pi^2} \frac{1}{2u} \Big((\frac{m^2}{2} + u) \sqrt{\frac{\frac{m^2}{2} + u}{\tilde{p}^2}} K_1 \left(\sqrt{(\frac{m^2}{2} + u)\tilde{p}^2} \right) \\ &\quad - (\frac{m^2}{2} - u) \sqrt{\frac{\frac{m^2}{2} - u}{\tilde{p}^2}} K_1 \left(\sqrt{(\frac{m^2}{2} - u)\tilde{p}^2} \right) \Big). \end{aligned} \tag{23}$$

For $\tilde{c} = 0 \Rightarrow u = \frac{m^2}{2}$ this yields exactly the result (15). With the expansion of the modified Bessel function we find for the $(\tilde{p}^2 \to 0)$-divergent part

$$\begin{aligned}\Delta\Sigma_{np\pm} &= \frac{g^2}{24\pi^2}\Big(\frac{1}{\tilde{p}^2} + \frac{1}{8u}(\frac{m^2}{2}+u)^2 \ln((\frac{m^2}{2}+u)\tilde{p}^2) \\ &\qquad -\frac{1}{8u}(\frac{m^2}{2}-u)^2 \ln((\frac{m^2}{2}-u)\tilde{p}^2) + \ldots\Big), \end{aligned} \tag{24}$$

where the dots denote the terms finite for $\tilde{p}^2 \to 0$. The logarithmic term is harmless, but we have to keep an eye on the $\frac{1}{\tilde{p}^2}$ term. We find that this term is invariant with respect to the choice of sign in (19). Therefore it is cancelled by the counterterm in the action only if we choose the positive sign in (19), so the counterterm reads

$$\delta\Gamma = +\int d^4x \phi(x) \frac{\tilde{c}}{\tilde{\partial}^2}\phi(x) \tag{25}$$

(note that $\tilde{\partial}^2 \Rightarrow -\tilde{k}^2$). The result of the planar part is obtained by multiplying (23) by 2 and taking the limes $\tilde{p} \to 0$.

$$\begin{aligned}\Delta\Sigma_{p\pm} &= \frac{g^2}{12\pi^2}\lim_{\Lambda\to\infty}\Big(\Lambda^2 - \frac{1}{8u}(\frac{m^2}{2}+u)^2 \ln\left(\frac{\Lambda^2}{\frac{m^2}{2}+u}\right) \\ &\qquad +\frac{1}{8u}(\frac{m^2}{2}-u)^2 \ln\left(\frac{\Lambda^2}{\frac{m^2}{2}-u}\right) + \ldots\Big), \end{aligned} \tag{26}$$

where again the finite terms are ignored. Thus, also in resummed field theory the planar one-loop correction of the two-point function can be absorbed in an ordinary mass renormalization of the theory.

4 Resummation in $U(1)$-noncommutative YM-theory?

In this section we try to sketch the idea of a resummation procedure for a gauge field model with BRS-symmetry [8]. We begin with a pure $U(1)$-noncommutative Yang-Mills (NCYM)-theory, which is described in Euclidean space at the classical level by

$$\Gamma^{(0)}_{INV} = \frac{1}{4}\int d^4x F_{\mu\nu} \star F_{\mu\nu}, \tag{27}$$

where the field strength $F_{\mu\nu}$ is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]_M, \quad [A,B]_M := A\star B - B\star A. \tag{28}$$

The enclosure of fermions can be done in the usual way [9], [10]. Scalar matter fields are treated in [11]. In order to allow a meaningful perturbation theory one has to use the BRS-quantization procedure [8], which implies the introduction of ghost fields $c, \bar{c}$ and a multiplier field B for the gauge fixing. One has the total action

$$\begin{aligned}\Gamma^{(0)} &= \Gamma^{(0)}_{INV} + \Gamma_{gf} + \Gamma_{matter} \\ &= \frac{1}{4}\int d^4x F_{\mu\nu}\star F_{\mu\nu} + \int d^4x (gB\star\partial_\mu A_\mu + \frac{\alpha}{2}B\star B - \bar{c}\star\partial_\mu D_\mu c) + \Gamma_{matter}, \end{aligned} \tag{29}$$

where $D_\mu := \partial_\mu - ig[A_\mu, .]_M$. The corresponding BRS-transformation is given by

$$\begin{aligned} sA_\mu &= D_\mu c, \qquad & sc &= ic \star c, \\ s\bar{c} &= B, \qquad & sB &= 0. \end{aligned} \tag{30}$$

Doing now perturbation theory at the one-loop level (including all contributions coming from the gluon, ghost, fermion etc. fields) one obtains the following problematic contribution to the vacuum polarization of the photon [9],

$$\Pi_{\mu\nu}(k) = \beta \frac{\tilde{k}_\mu \tilde{k}_\nu}{(\tilde{k}^2)^2} \tag{31}$$

(with β some numerical constant of order g^2), which represents the non-planar one-loop contribution. This is the well-known UV/IR-mixing term for non-supersymmetric NCYM-models. It is singular for $k_\mu \to 0$. The corresponding term for resummation would be

$$\delta\Gamma = \beta \int \frac{d^4k}{(2\pi)^4} A_\mu(k) A_\nu(-k) \frac{\tilde{k}_\mu \tilde{k}_\nu}{(\tilde{k}^2)^2}. \tag{32}$$

As observed in [11] it is gauge-invariant with respect to an infinitesimal Abelian gauge transformation, $\delta A_\mu = \partial_\mu \lambda$. But this is not the full story, one has to respect also BRS-invariance (30). In order to transform (32) in a BRS-invariant quantity we first define

$$\tilde{F} := \theta_{\mu\nu} F_{\mu\nu}, \quad \tilde{D}_\mu := \theta_{\mu\nu} D_\nu. \tag{33}$$

The idea is to replace (32) by (written in Euclidean coordinate space)

$$\delta\Gamma_{INV} = +\frac{\beta}{4} \int d^4x \tilde{F} \star \frac{1}{(\tilde{D}^2)^2} \star \tilde{F}. \tag{34}$$

Of course, $\frac{1}{(\tilde{D}^2)^2}$ is meant as power series in the gauge field A_μ, so we obtain an infinite set of nonlocal vertices (however, to each order in the gauge coupling g only a finite number of these vertices contribute). To lowest order in A_μ (32) and (34) are identical. Indeed, it is easy to show that $\frac{1}{(\tilde{D}^2)^2} \star X$ transforms covariantly if X does. For $X = \tilde{F}$ one has

$$\delta_\lambda \tilde{F} = i[\lambda, \tilde{F}]_M \Longrightarrow \quad \delta_\lambda\left(\frac{1}{(\tilde{D}^2)^2} \star \tilde{F}\right) = i[\lambda, \frac{1}{(\tilde{D}^2)^2} \star \tilde{F}]_M, \tag{35}$$

implying that (34) is BRS-invariant. In order to get a resummed gauge field model one generalizes now the calculation of the last section. The resummed action reads

$$\begin{aligned} \Gamma_R^{(0)} &= \int d^4x \Big(\frac{1}{4} F_{\mu\nu} \star F_{\mu\nu} + gB \star \partial_\mu A_\nu + \frac{\alpha}{2} B \star B - \bar{c} \star \partial_\mu D_\mu c \\ &\qquad + \frac{\beta}{4} \tilde{F} \star \frac{1}{(\tilde{D}^2)^2} \star \tilde{F} - \frac{\beta}{4} \tilde{F} \star \frac{1}{(\tilde{D}^2)^2} \star \tilde{F} \Big) + \Gamma_{matter} \end{aligned} \tag{36}$$

A similar ansatz for the solution of an analogous problem in high temperature QCD can be found in [12]. Taking only the bilinear part of (36) one can calculate the resummed U(1)-gauge field propagator as

$$\Delta_{\mu\nu\pm}(k) = -\frac{1}{k^2}\Big(g_{\mu\nu} - (1-\alpha)\frac{k_\mu k_\nu}{k^2} \mp 2\beta\frac{\tilde{k}_\mu \tilde{k}_\nu}{(\tilde{k}^2)^2(k^2 \pm \frac{2\beta}{\tilde{k}^2})}\Big). \tag{37}$$

The upper sign corresponds to the inclusion of the positive β-term in (36) in the propagator (treating the negative β-term as counterterm). One observes that the new term in the resummed propagator is independent of the gauge parameter α. Moreover, it is transversal due to $k_\mu \tilde{k}_\mu = 0$. The correct sign must be checked by explicit one-loop calculations [13]. Note that the resummation procedure for gauge theories involves also a resummation of the vertices due to the non-bilinear parts in the resummed term in (36).

5 Conclusion

In this short note we have reinvestigated the resummation procedure for scalar noncommutative quantum field theory. We have discussed the two possibilities of sign combination in the resummed action (18). It was verified by one-loop calculations that only

$$\Delta_+(k) = \frac{1}{k^2 + m^2 + \frac{\tilde{c}}{\tilde{k}^2}} \tag{38}$$

is useful in order to compensate the IR-singularity of $\frac{1}{\tilde{k}^2}$. Encouraged by works in thermal quantum field theory [12], [14] we have proposed a generalization for gauge field models. However, one has to stress that the relevant calculations have still to be done [13].

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A Integrals

First we need the complex Euclidean Gauss integral (α real)

$$\begin{aligned}\int d^4k e^{\pm i\alpha k^2 + ik\tilde{p}} &= \int d^4k e^{\pm i\alpha(k \pm \frac{\tilde{p}}{2\alpha})^2 \mp \frac{i\tilde{p}^2}{4\alpha}} = \\ &= \lim_{\varepsilon\to 0}\int d^4k' e^{-(\varepsilon \mp i\alpha)k'^2 \mp \frac{i\tilde{p}^2}{4\alpha}} = -\frac{\pi^2}{\alpha^2} e^{\mp\frac{i\tilde{p}^2}{4\alpha}}.\end{aligned} \tag{39}$$

With this we get for real $b > 0$, a real,

$$\int d^4k \frac{k^2}{k^2 + a \pm ib} e^{ik\tilde{p}} =$$

$$
\begin{aligned}
&= \int d^4k(\pm i)\int_0^\infty d\alpha(\pm i\frac{\partial}{\partial\alpha} + a \pm ib)e^{\pm i\alpha(k^2+a\pm ib)+ik\tilde{p}} \\
&= \pm i\int_0^\infty d\alpha(\pm i\frac{\partial}{\partial\alpha} + a \pm ib)\big(-\frac{\pi^2}{\alpha^2}e^{\mp\frac{i\tilde{p}^2}{4\alpha}\pm i\alpha(a\pm ib)}\big) \\
&= -(\pm i)\pi^2(a\pm ib)\lim_{\varepsilon\to 0}\int_0^\infty \frac{d\alpha}{\alpha^2}e^{-\frac{\varepsilon\pm i\tilde{p}^2}{4\alpha}-(b\mp ia)\alpha}. \qquad (40)
\end{aligned}
$$

Here we use $\int_0^\infty \frac{d\alpha}{\alpha^2}\exp(-u\alpha - v/(4\alpha)) = 4(\sqrt{u}/\sqrt{v})K_1(\sqrt{uv})$ for positive real part of u and v and find

$$
\begin{aligned}
&= -(\sqrt{\pm i})^2 4\pi^2(a\pm ib)\frac{\sqrt{b\mp ia}}{\sqrt{\pm i\tilde{p}^2}}K_1(\sqrt{\pm i\tilde{p}^2(b\mp ia)}) \\
&= -4\pi^2(a\pm ib)\sqrt{\frac{a\pm ib}{\tilde{p}^2}}K_1(\sqrt{(a\pm ib)\tilde{p}^2}). \qquad (41)
\end{aligned}
$$

Of course one has to be careful with respect to the sign of the roots, thus one should check the result (41) for special cases (e. g. $a > 0,\ b \to 0$) via ordinary (non-complex) Schwinger parametrization.

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